



Mark Scheme (Final)

October 2019

Pearson Edexcel IAL Mathematics
Pure Mathematics (1) (WMA11/01)

Question Number	Scheme	Marks
1.(a)	Sets $\frac{1}{2}r^2 \times 1.25 = 15 \Rightarrow r^2 = 24$ $\Rightarrow r = \sqrt{24}$ or $2\sqrt{6}$ (only)	M1 A1 (2)
(b)	Attempts $s = r\theta = 2\sqrt{6} \times 1.25$ Attempts $P = 2r + r\theta = 2 \times 2\sqrt{6} + 2\sqrt{6} \times 1.25$ $= \frac{13\sqrt{6}}{2}$ oe	M1 dM1 A1 (3) (5 marks)

(a)

M1 Uses $A = \frac{1}{2}r^2\theta$ in an attempt to find r

A1 $r = \sqrt{24}$ or $2\sqrt{6}$ only (oe) (isw after a correct answer is seen) Withhold the A if $r = \pm 2\sqrt{6}$ is given.

(b)

M1 Uses the formula $s = r\theta$ with their r and $\theta = 1.25$ in an attempt to find the arc length.

dM1 For applying $P = 2r + r\theta$ their r and $\theta = 1.25$ in an attempt to find the perimeter.

A1 $\frac{13\sqrt{6}}{2}$ (isw after a correct answer is seen). Accept $6.5\sqrt{13}$ (oe simplest forms)

Question Number	Scheme	Marks
2. (a)	$1.85 = 2a + b$ and $3.45 = 7a + b$ Solves simultaneously to get $a = 0.32, b = 1.21$ (oe)	M1 A1 dM1 A1 (4)
(b)	States 1.21 m or 121 cm (oe)	B1ft (1) (5 marks)

(a)

M1 For either $1.85 = 2a + b$ or $3.45 = 7a + b$

A1 For both $1.85 = 2a + b$ and $3.45 = 7a + b$

dM1 Solves simultaneously to get a value for a and a value for b

A1 $a = 0.32, b = 1.21$ or equivalent fractions. May be seen in the equation.

(b)

B1 ft States 1.21 m or 121 cm (oe including units). Correct answer or follow through on their **positive** b
Alt part (a)

M1 Attempts $\frac{3.45 - 1.85}{7 - 2}$ Allow from attempts at use of arithmetic series, or from incorrect indexing. So

E.g. $1.85 = 3a + b, 3.45 = 8a + b \Rightarrow a = \dots$ or $a_n = a + (n-1)d, a_1 = 1.85, a_6 = 3.45 \Rightarrow a = \dots$ gain this mark.

A1 $a = 0.32$ (may be called d if using AS)

dM1 Full correct method to find b E.g substitutes their $a = 0.32$ into either **correct** equation (with correct indexing), or in " $a_n = a + (n-1)d$ " finds " a " (=1.53) and expands to find $b = "a" - "d"$.

A1 $a = 0.32, b = 1.21$

Question Number	Scheme	Marks
3. (a)	$x^2 - 5x + 13 = (x - 2.5)^2 - 2.5^2 + 13 = (x - 2.5)^2 + 6.75$ <p>Coordinates $M = (2.5, 6.75)$</p>	M1 A1 A1 (3)
(b)	<p>Attempts the equation of l using their M $y = \frac{6.75}{2.5}x$ ($y = 2.7x$)</p> <p>Attempts to solve their $y = 2.7x$ with $y = x^2 - 5x + 13$</p> $\Rightarrow 2.7x = x^2 - 5x + 13 \Rightarrow x^2 - 7.7x + 13 = 0 \Rightarrow (x - 2.5)(x - 5.2) = 0$ <p style="text-align: right;">$x = 5.2$ oe</p> <p style="text-align: right;">Coordinates $N = (5.2, 14.04)$</p>	M1 M1 A1 dM1 A1 (5)
(c)	<p>States two of $y < x^2 - 5x + 13$, $y > 2.7x$, $0 \leq x < 2.5$</p> <p>States all three of $y < x^2 - 5x + 13$, $y > 2.7x$, $0 \leq x < 2.5$</p>	M1 A1ft (2)
		(10 marks)

(a)

M1 For attempting to complete the square. Look for $(x - 2.5)^2$

A1 $(x - 2.5)^2 + 6.75$ or for correctly extracting $x = 2.5$ as the x coordinate.

A1 $M = (2.5, 6.75)$ or $\left(\frac{5}{2}, \frac{27}{4}\right)$

(b)

M1 Uses their M to find an equation for l . Look for a correct attempt at the gradient, so $y = \frac{"6.75"}{"2.5"}x$ or $y - "6.75" = \frac{"6.75"}{"2.5"}(x - "2.5")$

M1 Depends on having made an attempt (not necessarily correct) to use O and M to find the equation of l . Attempts to solve their equation for l with $y = x^2 - 5x + 13$ Look for a full attempt leading to $x = \dots$

Correct answers following a correct simplified quadratic is fine for the method.

A1 $x = 5.2$ or equivalent such as $x = \frac{26}{5}$

dM1 Depends on second M mark. Substitutes their $x = 5.2$ into either equation to find $y = \dots$ If a y value follows an x value with no clearly incorrect working, then allow as an attempt.

A1 Coordinates $N = (5.2, 14.04)$ oe such as $\left(\frac{26}{5}, \frac{351}{25}\right)$. Allow this to be written separately.

(c)

M1 States any two of $y < x^2 - 5x + 13$, $y > "2.7"x$, $0 \leq x < b$ where $"2.5" \leq b \leq "5.2"$

Also allow the first two combined, or with loose inequalities, e.g $"2.7"x \leq y \leq x^2 - 5x + 13$

Also allow either \leq or $<$ on the left hand end of $0 \leq x < 2.5$ due to the y -axis.

Use of R instead of y is M0

A1ft States all three inequalities. Same conditions as above, follow through on their $2.7x$ from (b) and 2.5

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Allow part (a) to be attempted via finding minimum point

M1 For setting $\frac{dy}{dx} = ax + b = 0$ and finding a value for x , or for stating $x_{min} = -b/2a = \dots$

A1 $x = 2.5$

A1 $M = (2.5, 6.75)$

Note: Answer with no working score M0A0A0

Question Number	Scheme	Marks
4. (a)	Area $ABCD$ is $40 \text{ cm}^2 \Rightarrow 40 = 6 \times 10 \times \sin \theta$ oe $\sin \theta = \frac{2}{3} \Rightarrow \theta = 180^\circ - 41.8^\circ$ $\angle DAB = \text{awrt } 138.19^\circ$	M1 M1 A1 (3)
(b)	Attempts $DB^2 = 10^2 + 6^2 - 2 \times 10 \times 6 \cos "138.19^\circ"$ $DB = \text{awrt } 15.0 \text{ (cm)}$	M1 A1 (2) (5 marks)

- (a)
M1 Scored for a correct attempt at using the area of $ABCD$ is 40 cm^2
Score for $40 = 6 \times 10 \times \sin \theta$ or $20 = \frac{1}{2} \times 6 \times 10 \times \sin \theta$ where θ is one of the corner angles.
M1 Score for $\sin \theta = k \Rightarrow \theta = 180^\circ - \arcsin k$

A1 $\angle DAB = \text{awrt } 138.19^\circ$

(b)

M1 Attempts $DB^2 = 10^2 + 6^2 - 2 \times 10 \times 6 \cos "138.19^\circ"$ - allow if the angle used is acute as long as it is clearly their attempt at angle DAB . So allow use of 41.8° unless they have correctly found angle DAB and chosen the wrong one here.

A1 $DB = \text{awrt } 15.0(\text{cm})$ Accept 15 in place of 15.0. Allow from attempts using awrt 138°

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Alt for (a)

M1 Area $ABCD$ is $40 \text{ cm}^2 \Rightarrow h = \frac{40}{10} = \dots \Rightarrow \sin \angle ABC = \frac{"4"}{6}$ OR $\cos \angle ABC = \frac{"4"}{6}$ oe

Essentially this mark is for using the area together with an appropriate trig identity to form an equation in the sine or cosine of one of the angles of the parallelogram. Attempts finding " DX " where X is where the perpendicular to DC through A meets DC are possible.

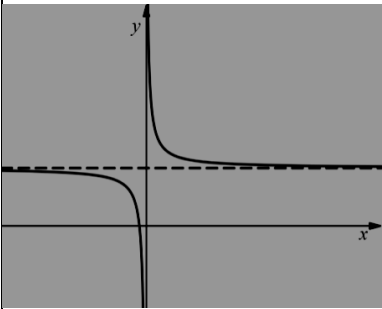
M1 $\angle DAB = 180^\circ - \arcsin\left(\frac{"4"}{6}\right) = \dots$ or may see $\angle DAB = 90^\circ + \arccos\left(\frac{"4"}{6}\right) = \dots$

This is for a complete correct method to find the angle DAB

A1 $\theta = \text{awrt } 138.19^\circ$

Question Number	Scheme	Marks
5.(a)	$\frac{dy}{dx} = \frac{1}{2}x^2 + 2x^{-\frac{1}{2}}$	M1A1 A1 (3)
(b)	$\left. \frac{dy}{dx} \right _{x=4} = \frac{1}{2} \times 4^2 + 2 \times \frac{1}{\sqrt{4}} = (9)$ Gradient of normal is $-\frac{1}{9}$ $y - \frac{11}{3} = -\frac{1}{9}(x - 4) \Rightarrow x + 9y - 37 = 0$	M1 dM1 M1 A1 (4) (7 marks)

- (a)
- M1 For reducing the power by one on any x term
- A1 Two correct terms which may be unsimplified (but ignore spurious extra terms like -15 for this mark)
- A1 Fully correct and simplified. Accept exact simplified equivalents. Eg $\frac{x^2}{2} + \frac{2}{\sqrt{x}}$
- Withhold the final A mark if “+c” is included.
- (b)
- M1 For substituting $x = 4$ into their $\frac{dy}{dx}$
- dM1 For the correct method of using the negative reciprocal to find the gradient of the normal.
- M1 For an attempt at finding the equation of the normal. It is for using a changed gradient and the point $\left(4, \frac{11}{3}\right)$ If the form $y = mx + c$ is used they must proceed to $c = \dots$
- A1 $x + 9y - 37 = 0$ or any (positive or negative) integer multiple thereof.
Accept with terms in a different order, but must include “=0”. ISW after a correct answer.

Question Number	Scheme	Marks
6. (a)	 <p>Shape</p> <p>States asymptote as $y = k$</p> <p>States intercept as $-\frac{4}{k}$</p>	<p>B1</p> <p>B1</p> <p>B1</p> <p>(3)</p>
(b)	$10 - 2x = \frac{4}{x} + k \Rightarrow 10x - 2x^2 = 4 + kx$ $\Rightarrow 2x^2 + (k - 10)x + 4 = 0$ <p>Attempts "$b^2 - 4ac = 0 \Rightarrow (k - 10)^2 - 4 \times 2 \times 4 = 0$</p> $k = 10 \pm 4\sqrt{2} \text{ oe}$	<p>M1</p> <p>A1</p> <p>M1</p> <p>M1 A1</p> <p>(5)</p> <p>(8 marks)</p>

(a)

B1 Correct shape for graph. Look for a $y = \frac{1}{x}$ curve translated in any direction. Be tolerant with slips of pen how

close the approach to asymptotes are, but the curve must not bend back on itself.

B1 Curve has horizontal asymptote above the x -axis with asymptote **stated** as $y = k$ on diagram or in text. (Do not accept just k marked on the axis for this mark.)

B1 Curve crosses (not just touches) the negative x -axis, with intercept marked or stated as $-\frac{4}{k}$

(b)

M1 Equates and attempts to multiply by x obtaining terms in x^2 , x and constant(s).

A1 $2x^2 + (k - 10)x + 4 = 0$ This may be implied by a correct a , b and c

M1 Attempts " $b^2 - 4ac = 0 \Rightarrow (k - 10)^2 - 4 \times 2 \times 4 = 0$ Withhold this mark if an inequality is applied.

M1 For a correct method of finding at least one value for k from this attempt at the discriminant which is quadratic in k . Allow this mark for attempts using inequalities rather than equality, e.g. $b^2 - 4ac < 0$

A1 $k = 10 \pm 4\sqrt{2}$ oe (Do not accept decimal approximations and do not isw if an inequality is later stated.)

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Alt for 6(b)

M1 Attempts to set $\frac{dy}{dx} = -2$ (usual rules for differentiation)

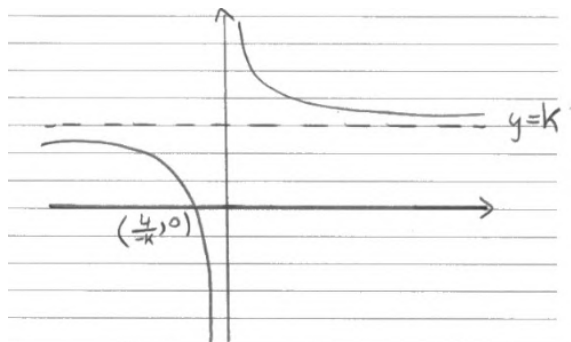
A1 Finds at least one value on the curve at which $\frac{dy}{dx} = -2 \Rightarrow -\frac{4}{x^2} = -2 \Rightarrow x = \pm\sqrt{2}$ (accept awrt ± 1.41)

dM1 Substitutes one of their x values into $y = 10 - 2x$ to find at least one y value. Eg $x = \sqrt{2}$, $y = 10 - 2\sqrt{2}$

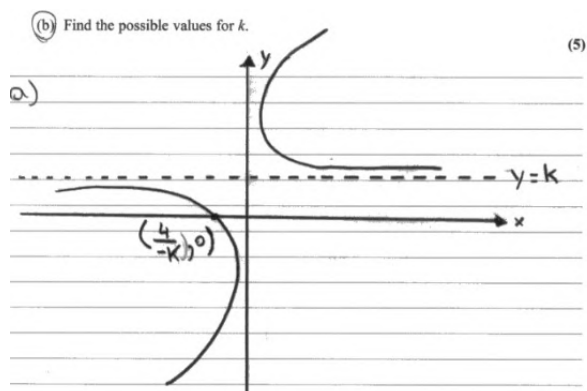
ddM1 Substitutes one of their (x, y) values into $y = \frac{4}{x} + k$ and proceeds to find one value for k .

A1 $k = 10 \pm 4\sqrt{2}$

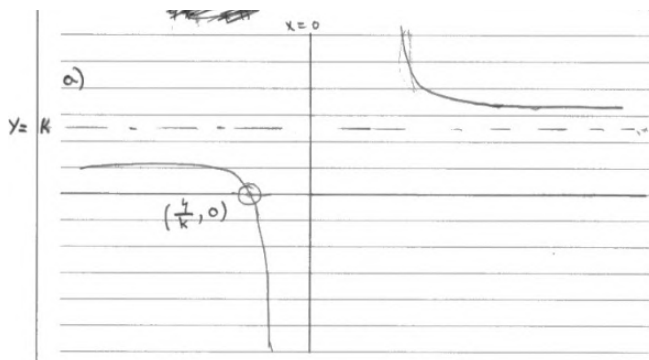
Examples for part (a)



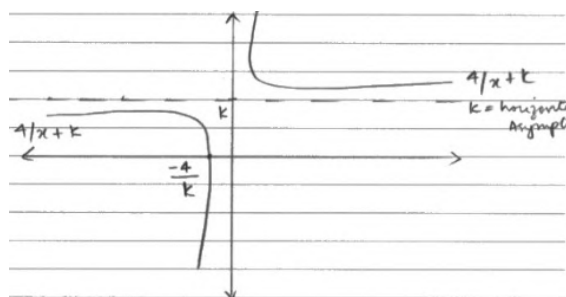
B1B1B1 Pen slip rather than turning back



B0B1B1 Turning back on itself



B1B1B0 Generous with a bit of gap between curve and asymptote



B0B0B1 All four approaches bend away from the asymptote

Question Number	Scheme	Marks
7.(a)	Attempts $\frac{dy}{dx} = 4x$ at $x = 2$ At $x = 2$ gradient of tangent = 8	M1 A1 (2)
(b)	$(y_Q =) 2(2+h)^2 + 5$ Gradient $PQ = \frac{\text{their } y_Q - 13}{2+h-2}$ $\left(= \frac{8h+2h^2}{h} \right) = 8+2h$	B1 M1 A1 (3)
(c)	States as $h \rightarrow 0$ Gradient $PQ \rightarrow 8 =$ Gradient of tangent	B1 (1) (6 marks)

(a)

M1 Attempts to find the value of $\frac{dy}{dx} = ax$, $a > 0$ at $x = 2$

A1 For 8. No need to state this is the gradient.

(b)

B1 $(y_Q =) 2(2+h)^2 + 5$

M1 Attempts $\pm \frac{y_Q - y_P}{x_Q - x_P}$ condoning slips, but must be a genuine attempt at y_Q

A1 Gradient is $8+2h$ (with no errors seen)

(c)

B1 States as $h \rightarrow 0$ Gradient $PQ \rightarrow 8 =$ Gradient of tangent (oe)

There should be reference to “limit” or “as h tends to 0” (words or symbols) and linked to part (a) (so same gradient, or showing the answers agree). But be generous with the explanation beyond these constraints.

Question Number	Scheme	Marks
8	$x - 6x^{\frac{1}{2}} + 4 = 0$ $x^{\frac{1}{2}} = 3 \pm \sqrt{5} \text{ oe}$ $x = (3 \pm \sqrt{5})^2 \Rightarrow x = 14 \pm 6\sqrt{5}$	M1 A1 M1 A1 A1 (5 marks)

M1 For attempting to solve an equation of the form $y^2 - 6y + 4 = 0$ by completing the square or quadratic formula to reach at least one solution. There must be some working shown for this mark to be awarded, accept as a minimum identifying $y = x^{\frac{1}{2}}$ and writing the quadratic in y before solutions.

A1 $\left(x^{\frac{1}{2}}\right) = 3 \pm \sqrt{5}$ Both required (though one may be later rejected) but need not be simplified, so accept $\frac{6 \pm 2\sqrt{5}}{2}$

M1 For attempting to square a solution of the form $p \pm q\sqrt{r}$ with 2 (out of 4) correct terms (may be implied by correct answers for their terms, but must have seen at least one solution for $x^{\frac{1}{2}}$)

A1 $x = 14 + 6\sqrt{5}$ **or** $x = 14 - 6\sqrt{5}$ as an answer Accept equivalents for this mark.

A1 $x = 14 + 6\sqrt{5}$ **and** $x = 14 - 6\sqrt{5}$ as answers, must be simplified.

Special Case: For candidates who show no initial working and write $x^{\frac{1}{2}} = 3 \pm \sqrt{5}$ as their first step, M0A0M1A1A1 is possible if they go on to achieve correct answers

Question Number	Scheme	Marks
8 Alt	$x + 4 = 6x^{\frac{1}{2}}$ $(x + 4)^2 = 36x$ $x^2 - 28x + 16 = 0 \Rightarrow (x - 14)^2 = 180 \Rightarrow x = 14 \pm \sqrt{180} \Rightarrow x = 14 \pm 6\sqrt{5}$	M1 A1 M1 A1 A1 (5)

M1 Isolates the square root term and squares both sides.

A1 Correct squared expression, $(x + 4)^2$ need not be expanded (as in scheme).

M1 Expands and solves the quadratic in x Note that candidates who square term by term will score no marks.

A1A1 As main scheme. Note for the final A both solutions must be fully simplified.

Question Number	Scheme	Marks
9. (a)	24π	B1 (1)
(b)	$(18\pi, -1)$	B1ft (1)
(c)(i)	$-12\pi - \alpha$	B1 ft
(ii)	$6\pi - \alpha$	B1 ft (2)
		(4 marks)

(a) Do not allow the mark if an inequality or coordinates are given, but you may assume if $0 < x < p$ or $(p, 0)$ is given that their period is p for the purposes of the remaining marks.

See above

Note that (b) and (c) are for (correct or) follow through and hence

(b) is scored for $\left(\frac{3p}{4}, -1\right)$ -- this may be seen on the graph but take clearly labelled part (b) as precedent.

Follow through on $p \neq 2\pi$

(c) (i) is scored for $-\frac{1}{2}p - \alpha$

(ii) is scored for $\frac{1}{4}p - \alpha$

where p is their period stated in (a) (which may be 2π)

For answers in degrees penalise the first mark due only, but allow if degree symbol is missing.

For reference (a) 4320° (b) $(3240^\circ, -1)$ (c)(i) $-2160^\circ - \alpha$ (ii) $1080^\circ - \alpha$

Note for example that (a) $0 < x < 4320^\circ$ (b) $(3240^\circ, -1)$ (c)(i) $-2160^\circ - \alpha$ (ii) $1080^\circ - \alpha$ would score B0 B0 B1ft B1ft

Question Number	Scheme	Marks
10 (a)	$f(x) \leq 0 \Rightarrow x \leq -\frac{5}{2}, \quad x = 3$	M1 A1

Question Number	Scheme	Marks
(b)	$f(x) = (2x+5)(x-3)^2 = (2x+5)(x^2 - 6x + 9)$ $= 2x^3 - 12x^2 + 18x + 5x^2 - 30x + 45$ $= 2x^3 - 7x^2 - 12x + 45$	<p>(2)</p> <p>M1</p> <p>M1</p> <p>A1</p> <p>(3)</p>
(c)	<p>(i) $P(0,45)$</p> <p>(ii) Gradient = -12</p>	<p>B1ft</p> <p>B1ft</p> <p>(2)</p>
(d)	<p>(i) $g(x) = (2(x-2)+5)(x-2-3)^2 = (2x+1)(x-5)^2$</p> <p>(ii) 25</p>	<p>M1 A1</p> <p>B1</p> <p>(3)</p> <p>(10 marks)</p>

(a)

M1 For either $x \leq -\frac{5}{2}$ or $x = 3$ Condone for this mark $x < -\frac{5}{2}$

A1 For both $x \leq -\frac{5}{2}$ and $x = 3$ Accept answers given in set notation. Accept with “and” or “or” between – it is for both correct inequalities.

Note: Mark the final answer. Answers such as $-\frac{5}{2} \leq x \leq 3$ or $3 \leq x \leq -\frac{5}{2}$ are M0A0.

(b)

M1 Attempts to multiply two of the brackets together, achieving x^2 , x and constant terms.

M1 Multiplies the result by the third bracket to reach a four term cubic expression (not necessarily simplified).

A1 $2x^3 - 7x^2 - 12x + 45$ Ignore any reference to “=0” or “<0” etc for this mark.

(c)(i)

B1ft $P(0,45)$ following through on their 'd'. Do not accept e.g. (45,0) or just 45.

(c)(ii)

B1 ft Gradient = -12 following through on their 'c'

(d)(i)

M1 Attempts to replace x with $(x-2)$ in $(2x+5)(x-3)^2$ or in their expansion of this.

Allow M1 for one correct bracket if no incorrect working is seen (bod). Condone invisible brackets

A1 $g(x) = (2x+1)(x-5)^2$ If substituting into the expand form, must factorise correctly to achieve A1.

(d)(ii)

B1 25 Accept (0,25).

Question Number	Scheme	Marks

11. (a)	Attempts $y - \frac{32}{3} = 5(x - 4) \Rightarrow y = 5x - \frac{28}{3}$	M1 A1 (2)
(b)	$f''(x) = \frac{4}{\sqrt{x}} - 3 \Rightarrow f'(x) = 8x^{\frac{1}{2}} - 3x + k$ Substitutes $x = 4, f'(x) = 5 \Rightarrow k = 1$ $f'(x) = 8x^{\frac{1}{2}} - 3x + 1 \Rightarrow f(x) = \frac{16}{3}x^{\frac{3}{2}} - \frac{3}{2}x^2 + x + d$ Substitutes $x = 4, f(x) = \frac{32}{3} \Rightarrow d = -12$ $f(x) = \frac{16}{3}x^{\frac{3}{2}} - \frac{3}{2}x^2 + x - 12$	<div style="display: flex; align-items: center;"> <div style="border: 1px solid black; width: 20px; height: 100px; margin-right: 5px;"></div> <div style="display: flex; flex-direction: column; gap: 5px;"> M1 A1 dM1 A1 dM1 A1 ddM1 A1 </div> </div> (8) (10 marks)

(a)

M1 Uses gradient of 5 at point $P\left(4, \frac{32}{3}\right)$ to form tangent. For example, $y - \frac{32}{3} = 5(x - 4)$

A1 $y = 5x - \frac{28}{3}$ Accept recurring decimal, but $y = 5x - 9.33$ is A0.

(b)

M1 Attempts to integrate $\frac{4}{\sqrt{x}} - 3$ with one **index** correct

A1 $(f'(x)) = \frac{4}{\sqrt{x}} - 3 \rightarrow 8x^{\frac{1}{2}} - 3x + k$ with or without the $+ k$

dM1 Substitutes $x = 4, f'(x) = 5$ into an integrated form (with $+ k$) and proceeds to find the value of k

A1 $f'(x) = 8x^{\frac{1}{2}} - 3x + 1$ which may be implied (allow if $k = 1$ is found following a correct integral with k)

dM1 Dependent upon the first M. It is for integrating 'again' with one **term** correct

A1 $f(x) = \frac{16}{3}x^{\frac{3}{2}} - \frac{3}{2}x^2 + kx + d$ following through on their $k \neq 0$ (which may be a letter or number) and with or without d (Both constants may be called c for this mark.)

ddM1 Dependent upon 1st and third M's (ie having attempted to integrate twice), **and** both " k " and " d " must have been added and processed correctly. Note " $cx + c$ " will score M0 as this is not a correct process for both constant

This mark is scored for using $x = 4, f(x) = \frac{32}{3}$ in an attempt to find ' d '

A1 $f(x) = \frac{16}{3}x^{\frac{3}{2}} - \frac{3}{2}x^2 + x - 12$ oe $(x\sqrt{x}$ instead of $x^{\frac{3}{2}}$ is fine)